## Comments on I1-branes

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ABSTRACT: We explore the supergravity solution of D5-branes intersecting as an I1-brane. In a suitable near-horizon limit the geometry is in qualitative agreement with that found in the microscopic open-string analysis as well as the NS5-brane analysis of Itzhaki, Kutasov and Seiberg. In particular, the $\operatorname{ISO}(1,1)$ Lorentz symmetry of the intersection domain is enhanced to $\operatorname{ISO}(1,2)$. The discussion is generalised to the T-dual configuration of a D4-brane intersecting a D6-brane. In this case the $\operatorname{ISO}(1,1)$ symmetry is not enhanced. This is true both in the supergravity approximation to the weakly coupled string theory and to the M-theory limit.

Keywords: Brane Dynamics in Gauge Theories, Intersecting branes models.

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## 1. Introduction

Gauge anomalies play a key role in the structure of quantum field theory and string theory. They also enter into curious phenomena in some brane configurations. It was recently observed in [1] and independently in [2] that close to the $1+1$ intersection region of intersecting five branes (an I1-brane) Poincare symmetry is enhanced from $\operatorname{ISO}(1,1)$ to $I S O(1,2)$. This was deduced in two ways. The first was a microscopic weakly-coupled open string analysis, involving anomaly inflow from the branes to the intersecting region. The second involved the supergravity limit of the S-dual brane configuration (i.e. NS5branes) in the near horizon limit. In fact the algebra of the enhanced supersymmetries is presented in [2]. In this note we analyse the supergravity limit of the branes directly in their D5-brane description. In this case special care is needed in taking the near horizon limit. Although the supergravity solution is applicable only in the large $N$ limit, where the number of branes in both stacks is large, and in a restricted range of the radial coordinates, the general picture that symmetry is enhanced is unchanged.

Then we move on to studying the T-dual configuration, namely a stack of D6's intersecting another stack of D4's in $1+1$ dimensions. Following the analysis in [1] we find that in this case there is no symmetry enhancement in both the gauge theory limit and the supergravity limit. In the gauge theory limit the anomaly inflow mechanism is still at work so that the chiral fermions are still displaced from the intersection region. However they are pushed into the bulk of D 6 and D 4 , respectively, with different radial dependences. This spoils the level-rank duality [3] argument so that Poincare symmetry enhancement is absent. This is confirmed in the supergravity limit where we find that angle deficits develop
in the near horizon limit which prevents symmetry enhancement. We will note that this is also true of the M-theory description.

The organisation of this paper is as follows: section 2 gives an overview of the results of [1]. Section 3 discusses the supergravity interpretation of the intersecting D5's. The generalisation to the D6-D4 system is described in section 4. Further comments are made in the last section, including a discussion of the M-theory limit of the D4/D6 system.

## 2. Overview of I-brane dynamics

Consider a BPS configuration of orthogonally intersecting branes, such that the number of relatively transverse dimensions is a multiple of four and that the intersection domain has $4 k+2$ dimensions. The massless spectrum consists chiral fermions that arise from open strings connecting a brane from each stack of intersecting branes. They give rise to gauge and gravitational anomalies from the perspective of the field theory in the intersection domain. This anomaly is cancelled by anomaly inflow from the rest of the brane (4), regarded here as the bulk. This mechanism implies that the D-brane world-volume action contains Chern-Simons terms of the form

$$
\begin{equation*}
\mu \int_{M_{D_{p}}} C \wedge c h_{N}(F) \sqrt{\hat{A}(R)}, \tag{2.1}
\end{equation*}
$$

where $C$ refers to the RR-forms, $c h_{r}(F)=\operatorname{Tr}_{r}\left(\exp \left(\frac{i F}{2 \pi}\right)\right)$ and $\hat{A}(R)$ is the A-roof genus. The integral picks out the forms such that the wedge product is a $p+1$ form on the $\mathrm{D} p$ brane world-volume. This mechanism played a crucial role in the microscopic analysis in [1], which we will now briefly review.

### 2.1 Open string perspective of intersecting D5 branes

Consider $N_{1}$ D5 branes intersecting another set of $N_{2}$ D5's orthogonally over $1+1$ dimensions. Suppose the first set lies along $x^{\mu}, \mu \in\{0,1,2,3,4,5\}$ and the second set along $\mu \in\{0,1,6,7,8,9\}$. There are altogether 8 relatively transverse directions. The system preserves a quarter supersymmetries i.e. eight supercharges.

$$
\begin{equation*}
\Gamma^{012345} \epsilon_{R}=\epsilon_{L}, \quad \Gamma^{016789} \epsilon_{R}=\epsilon_{L} . \tag{2.2}
\end{equation*}
$$

In the low energy limit the $5-5^{\prime}$ open strings essentially live in the intersection region and the field theory of interest is $1+1$ dimensional. From the perspective of the $1+1$ dimensional theory, the supercharges preserved are chiral satisfying

$$
\begin{equation*}
\Gamma^{01} \epsilon_{L, R}=\epsilon_{L, R} . \tag{2.3}
\end{equation*}
$$

The $5-5^{\prime}$ sector also contains massless chiral fermions. They originate from the RR zero modes along $x^{0}$ and $x^{1}$. After GSO projection we are left with one chiral fermion in the representation $\left(N_{1}, \bar{N}_{2}\right)$ of the gauge group $\mathrm{U}\left(N_{1}\right) \times \mathrm{U}\left(N_{2}\right)$ and so the field theory in the intersection is anomalous. However the theory including the intersection and bulk is anomaly free as discussed earlier. The total tree-level Lagrangian for the whole system is a
sum of the kinetic terms of the gauge fields and the chiral fermion and the Chern-Simons terms. The fermions are then integrated out to give a non-local action. The low energy limit is taken such that only the S -waves in the two 3 -spheres in the bulk of each of the two sets of branes are included.

The resulting action is quadratic in the fields and is explicitly anomaly free. Equations of motion obtained from this effective action describe wave functions for the gauge fields that are displaced away from the intersection region. For the simple case where $N_{1}=N_{2}=$ 1 , the solution is (1]

$$
\begin{equation*}
F_{u_{i} \pm}^{(i)}=\frac{h_{ \pm}}{u_{i}^{3}} e^{ \pm \frac{g_{i}^{2}}{u_{i}^{2}}}, \tag{2.4}
\end{equation*}
$$

where $i \in\{1,2\} . F^{(i)}=d A^{(i)}$ are the gauge field strengths of the respective branes with effective gauge coupling $g_{i} \sim g_{s} l_{s}^{2}$, and $u_{i}$ are the two radial directions away from the intersection region into the bulk of the branes. This implies that the chiral fermion is also displaced away from the intersection region. This general picture is not altered even if more coincident branes are considered such that the gauge theory in the intersection becomes a non-abelian theory. In general, where there are $N_{1}$ and $N_{2}$ branes in each stack respectively, we are left with a $\operatorname{SU}\left(N_{1}\right)_{N_{2}}$ Chern-Simons theory at $u_{1} \sim g_{1} \sqrt{N_{2}}$ and another $\operatorname{SU}\left(N_{2}\right)_{N_{1}}$ Chern-Simons theory at $u_{2} \sim g_{2} \sqrt{N_{1}}$ which, by level-rank duality [3], are the same. This level-rank duality is related to the modular invariance of the torus of the gravity dual [2]. Close to the intersection region the two radial directions cannot be distinguished. Poincare symmetry is enhanced from $\operatorname{ISO}(1,1)$ to $\operatorname{ISO}(1,2)$ and the number of conserved supercharges is also enhanced from 8 to 16 .

### 2.2 Closed string perspective

The same system can also be analysed by considering it as a black-brane solution to type IIB supergravity in the near horizon limit. It happens that for intersecting branes with exactly eight relatively transverse dimensions, a fully localised solution is known. Moreover it is actually more convenient to analyse the S-dual configuration. This is because the supergravity solution for the D-branes has a non-constant dilaton which grows indefinitely as we move away from the branes. As a result it ceases to be a good approximation away from the intersection. More importantly, in the D-brane description the number of branes $N_{1}$ and $N_{2}$ has to be large in order that the solution has a small enough curvature to be valid as a supergravity approximation. These constrain severely the regime where the solution is valid. We will return to D5-brane description in section 3. On the other hand switching to the S-dual picture with intersecting NS5-branes, the solution turns out to be an exact background of a conformal world-sheet theory and so the geometry obtained is not restricted to the supergravity approximation. The solution is valid for all $N_{1}, N_{2}>1$. Following the notation in [1], the metric of the configuration is given by

$$
\begin{equation*}
d s^{2}=-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+f_{1}(v)\left(d v^{2}+v^{2} d \Omega_{v}^{2}\right)+f_{2}(u)\left(d u^{2}+u^{2} d \Omega_{u}^{2}\right), \tag{2.5}
\end{equation*}
$$

where

$$
\begin{equation*}
y=\left(x^{2}, x^{3}, x^{4}, x^{5}\right), \quad z=\left(x^{6}, x^{7}, x^{8}, x^{9}\right), \quad \Phi=\Phi_{1}\left(y_{2}\right)+\Phi_{2}\left(y_{1}\right), \tag{2.6}
\end{equation*}
$$

and

$$
\begin{align*}
& e^{2\left(\Phi_{1}-\Phi_{1}(\infty)\right)}=f_{1}(v=|z|)=1+\sum_{n=1}^{N_{1}} \frac{l_{s}^{2}}{\left|z-z_{n}\right|^{2}}, \\
& e^{2\left(\Phi_{2}-\Phi_{2}(\infty)\right)}=f_{2}(u=|y|)=1+\sum_{n=1}^{N_{2}} \frac{l_{s}^{2}}{\left|y-y_{n}\right|^{2}} . \tag{2.7}
\end{align*}
$$

The $y_{n}$ 's and $z_{n}$ 's give the position of the branes. In the case under consideration $y_{n}=$ $z_{n}=0$. The near horizon limit is obtained by keeping $v / \exp \left(\Phi_{1}(\infty)\right)$ and $u / \exp \left(\Phi_{2}(\infty)\right)$ fixed while letting $\exp \left(\phi_{1}(\infty)\right), \exp \left(\phi_{2}(\infty)\right) \rightarrow 0$. This essentially means that we can drop the constant in front of the term $l_{s}^{2} /\left|z-z_{n}\right|^{2}$,

$$
\begin{equation*}
e^{2\left(\Phi_{1}-\Phi_{1}(\infty)\right)}=f_{1}(v=|z|) \sim \frac{N_{1} l_{s}^{2}}{\left|z-z_{n}\right|^{2}}, \tag{2.8}
\end{equation*}
$$

and similarly for $\exp \left(2\left(\Phi_{2}-\Phi_{2}(\infty)\right)\right)$. Making a change of coordinates

$$
\begin{equation*}
\phi_{1}=\sqrt{2 k_{1}} \ln v, \quad \phi_{2}=\sqrt{2 k_{2}} \ln u \tag{2.9}
\end{equation*}
$$

where $2 k_{i}=N_{i} l_{s}^{2}$, the resulting metric is

$$
\begin{equation*}
d s^{2}=-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+d \phi_{1}^{2}+d \phi_{2}^{2}+2 k_{2} d \Omega_{u}^{2}+2 k_{1} d \Omega_{v}^{2} . \tag{2.10}
\end{equation*}
$$

The metric looks flat in $x^{0}, x^{1}, \phi_{1}, \phi_{2}$. However the dilaton depends on $\phi_{1}, \phi_{2}$ and so the background is not invariant under general $\operatorname{ISO}(1,3)$ rotations. Consider a further coordinate change given by [1], 2]

$$
\begin{equation*}
Q \phi=Q_{1} \phi_{1}+Q_{2} \phi_{2}, \quad Q x^{2}=Q_{2} \phi_{1}-Q_{1} \phi_{2}, \tag{2.11}
\end{equation*}
$$

where

$$
\begin{equation*}
Q_{i}=\sqrt{\frac{2}{k_{i}}}, \quad Q=\sqrt{\frac{2}{k}}, \quad \frac{1}{k}=\frac{1}{k_{1}}+\frac{1}{k_{2}} . \tag{2.12}
\end{equation*}
$$

Then we end up with the same metric but now the dilaton depends only on $\phi$ and so the background is invariant under general $\operatorname{ISO}(1,2)$ rotations. The near horizon geometry has an enhanced Poincare symmetry as if an extra dimension has grown out from the intersection region as in the weak coupling description.

## 3. The D5 supergravity description

While the supergravity solution for the intersecting D5-brane configuration is only valid in a particular region in coordinate space and within a smaller regime of the couplings, it is nevertheless interesting to take a closer look at it. The fully localised solution for D5's intersecting over $1+1$ dimensions is given (in string frame) by

$$
\begin{array}{rlrl}
d s^{2} & =\left(H_{1} H_{2}\right)^{-\frac{1}{2}}\left(-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+H_{1}\left(d u^{2}+u^{2} d \Omega_{u}^{2}\right)+H_{2}\left(d v^{2}+v^{2} d \Omega_{v}^{2}\right)\right), \\
e^{(\Phi)} & =e^{\left(\Phi_{1}\right)} e^{\left(\Phi_{2}\right)}, & e^{\left(\Phi_{i}\right)}=H_{i}^{-\frac{1}{2}}, \\
H_{1} & =1+\frac{d_{5} g_{5}^{2} N_{1}}{u^{2}}, & H_{2}=1+\frac{d_{5} g_{5}^{2} N_{2}}{v^{2}}, \\
g_{5}^{2} & =(2 \pi)^{3} g_{s} \alpha^{\prime}, & & \tag{3.1}
\end{array}
$$

where $d_{5}$ is some constant whose value is defined in [6]. As in AdS/CFT we need to take the low energy limit which is the near horizon limit from the supergravity perspective. The Maldacena limit is however not the appropriate limit to be taken here. In the Maldacena limit where we let $\alpha^{\prime} \rightarrow 0$ while keeping $g_{5}^{2}$ and $u / \alpha^{\prime}$ and $v / \alpha^{\prime}$ fixed [6], both the string coupling and $d s^{2}$ in units of string length tend to zero. In order to keep these two quantities finite while taking the near horizon limit, we keep instead $U=u / \sqrt{\alpha^{\prime}}, V=v / \sqrt{\alpha^{\prime}}$ and $g_{s} \alpha^{\prime}$ fixed. This indeed is related to the limit taken for the intersecting NS5 case by S-duality. Then the metric becomes

$$
\begin{align*}
e^{\Phi} & =\frac{U V}{d_{5} \sqrt{N_{1} N_{2}}}  \tag{3.2}\\
d s^{2} & =\alpha^{\prime}\left(\frac{U V}{d_{5} g_{5}^{2} \sqrt{N_{1} N_{2}}}\right)\left(-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+d \phi_{1}^{2}+d \phi_{2}^{2}+d_{5} g_{5}^{2}\left(N_{1} d \Omega_{u}^{2}+N_{2} d \Omega_{v}^{2}\right)\right)
\end{align*}
$$

where

$$
\begin{equation*}
\phi_{1}=\sqrt{g_{5}^{2} N_{1} d_{5}} \ln U, \quad \phi_{2}=\sqrt{g_{5}^{2} N_{2} d_{5}} \ln V . \tag{3.3}
\end{equation*}
$$

Now we perform the same change of coordinates as in (2.9)

$$
\begin{equation*}
\sqrt{\frac{1}{N}} \Omega=\frac{\phi_{1}}{\sqrt{N_{1}}}+\frac{\phi_{2}}{\sqrt{N_{2}}}, \quad \sqrt{\frac{1}{N}} x^{2}=\frac{\phi_{1}}{\sqrt{N_{2}}}-\frac{\phi_{2}}{\sqrt{N_{1}}} \tag{3.4}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{1}{N}=\frac{1}{N_{1}}+\frac{1}{N_{2}} . \tag{3.5}
\end{equation*}
$$

The solution reduces to

$$
\begin{gather*}
d s^{2}=\alpha^{\prime}\left(\frac{e^{\Omega / \sqrt{g_{5}^{2} d_{5} N}}}{d_{5} g_{5}^{2} \sqrt{N_{1} N_{2}}}\right)\left(-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+\left(d x^{2}\right)^{2}+d \Omega^{2}+d_{5} g_{5}^{2}\left(N_{1} d \Omega_{u}^{2}+N_{2} d \Omega_{v}^{2}\right)\right) \\
e^{\Phi}=\frac{e^{\Omega / \sqrt{g_{5}^{2} d_{5} N}}}{d_{5} \sqrt{N_{1} N_{2}}} \tag{3.6}
\end{gather*}
$$

So we see that the solution exhibits enhanced Poincare symmetry, just as it did in the regimes considered in [1]

The solution is a supergravity approximation. Therefore it is only valid in the region where both the string coupling and the curvature are small. The string coupling is small when $U V \ll \sqrt{N_{1} N_{2}}$. Beyond that we have to go over to the S-dual picture. The Ricci scalar of the metric is approximately $\sim 1 / U V$ and this is small for $U V \gg 1$. Beyond that we need to go over to the gauge theory picture.

## 4. The D6-D4 system

It is interesting to T-dualise the intersecting D5-branes along one of the relatively transverse directions to obtain the D6-D4 system. The branes again intersect over $1+1$ dimensions and in the same way there is a chiral fermion in the intersection region, whose chiral anomalies have to be cancelled by the anomaly inflow mechanism via coupling with the bulk fields through the Chern-Simons terms. However, the symmetry enhancement observed in the intersection region in the system of 5-branes does not occur in the T-dual picture. This can be seen both from weak coupling gauge theory analysis and from the supergravity limit.

### 4.1 Supergravity solution of D6-D4 system

System of intersecting branes having eight relatively transverse directions without any totally transverse directions, is a special case where a completely localised supergravity solution can be found [7. 8]. Suppose the D4 is aligned along $x^{1,2,3,4}$ and D6 along $x^{1,5,6,7,8,9}$. The supergravity solution of the system is given by

$$
\begin{equation*}
d s^{2}=H_{6}^{-\frac{1}{2}} H_{4}^{-\frac{1}{2}}\left\{-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+H_{4}\left(d u^{2}+u^{2} d \Omega_{5}^{2}\right)+H_{6}\left(d v^{2}+v^{2} d \Omega_{2}^{2}\right)\right\} \tag{4.1}
\end{equation*}
$$

where

$$
\begin{gather*}
z=\left\{x^{2,3,4}\right\}, \quad y=\left\{x^{5,6,7,8,9}\right\}, \quad u=|y|, \quad v=|z|, \\
H_{4}=1+\frac{d_{4} g_{s} l_{s}^{3} N_{4}}{u^{3}}, \quad H_{6}=1+\frac{d_{6} g_{s} l_{s} N_{6}}{v}, \quad e^{\Phi}=H_{6}^{-\frac{3}{4}} H_{4}^{-\frac{1}{4}} \tag{4.2}
\end{gather*}
$$

This solution is valid in the small dilaton limit. As we move away from the branes the dilaton grows and the 11th dimension becomes important. The M-theory picture would then be the appropriate description in which the D6-brane becomes a Kaluza-Klein monopole and the D4-brane becomes an M5 brane wrapping on a circle. Consider first the perturbative closed string limit where the dilaton is small. We take the near horizon limit as in the case for the D5-branes by sending $\alpha^{\prime}$ to zero while keeping the ratio $U=r / \sqrt{\alpha^{\prime}}$ fixed. In this limit we can drop the constant, 1 , in $H_{4}$ and $H_{6}{ }^{1}$. Defining $k_{1}=d_{6} g_{s} l_{s} N_{6}$ and $k_{2}=d_{4} g_{s} l_{s}^{3} N_{4}$, the metric becomes

$$
\begin{equation*}
d s^{2}=H_{6}^{-\frac{1}{2}} H_{4}^{-\frac{1}{2}}\left\{-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+\frac{k_{2}}{u^{3}} d u^{2}+\frac{k_{2}}{u} d \Omega_{5}^{2}+\frac{k_{1}}{v} d v^{2}+k_{1} v d \Omega_{2}^{2}\right\} \tag{4.3}
\end{equation*}
$$

Under a change of coordinates

$$
\begin{equation*}
\phi_{1}=2 \sqrt{k_{1} v}, \quad \phi_{2}=-2 \sqrt{\frac{k_{2}}{u}} \tag{4.4}
\end{equation*}
$$

the metric can be written as

$$
\begin{equation*}
d s^{2}=H_{6}^{-\frac{1}{2}} H_{4}^{-\frac{1}{2}}\left\{-\left(d x^{0}\right)^{2}+\left(d x^{1}\right)^{2}+d \phi_{2}^{2}+\frac{\phi_{2}^{2}}{4} d \Omega_{5}^{2}+d \phi_{1}^{2}+\frac{\phi_{1}^{2}}{4} d \Omega_{2}^{2}\right\} \tag{4.5}
\end{equation*}
$$

The part with $d \phi_{2}^{2}+\frac{\phi_{2}^{2}}{4} d \Omega_{5}^{2}$ is almost a flat metric except the factor of 4 in the second term, which cannot be scaled away by rescaling $\phi_{2}$. This gives a space with an angle deficit. This applies also to $d \phi_{1}^{2}+\frac{\phi_{1}^{2}}{4} d \Omega_{2}^{2}$. Therefore the space does not exhibit enhanced Poincare symmetry in the near horizon limit.

### 4.2 Gauge field theory analysis

A similar analysis as for the intersecting D5 case is carried out here. The calculation is essentially the same. Basically we need to solve the equations of motion of the effective

[^0]Lagrangian after integrating out the chiral fermion that resides in the intersection region. The effective Lagrangian is similar to that of the intersecting D5 case, except for some slight modifications. Taking only one D6 and one D4, applying again the S-wave approximation where we ignore all angular dependence in the brane bulk, we have

$$
\begin{align*}
g_{\mathrm{Dp}}^{2} & =\frac{g_{s}^{2}}{\left(2 \pi \alpha^{\prime}\right) \mu_{p}},  \tag{4.6}\\
\mu_{p}^{2} & =\frac{\pi}{\kappa_{10}^{2}}\left(4 \pi^{2} \alpha^{\prime}\right)^{3-p}, \\
L_{\text {total }} & =L_{\mathrm{kin}}+L_{\mathrm{ferm}}+L_{\mathrm{cs}}+L_{\mathrm{mix}}, \\
L_{\mathrm{d} 6 \mathrm{kin}} & =\frac{V_{S^{5}}}{g_{\mathrm{D} 6}^{2}} \int d u u^{4}\left[\frac{1}{2}\left(F_{+-}^{(6)}\right)^{2}-\left(F_{+u}^{(6)} F_{-u}^{(6)}\right)\right], \\
L_{\mathrm{d} 4 \mathrm{kin}} & =\frac{V_{S^{2}}}{g_{\mathrm{D} 4}^{2}} \int d v v^{2}\left[\frac{1}{2}\left(F_{+-}^{(4)}\right)^{2}-\left(F_{+v}^{(4)} F_{-v}^{(4)}\right)\right], \\
L_{\mathrm{ferm}} & =\left(A_{+}^{(6)}(0)-A_{+}^{(4)}(0)\right) \frac{\partial_{-}}{\partial_{+}}\left(A_{+}^{(6)}(0)-A_{+}^{(4)}(0)\right)-\left(A_{+}^{(6)}(0)-A_{+}^{(4)}(0)\right)\left(A_{-}^{(6)}(0)-A_{-}^{(4)}(0)\right) .
\end{align*}
$$

The relevant Chern-Simons coupling to the bulk fields are

$$
\begin{equation*}
\int_{M_{6+1}} \frac{1}{\mu_{6}} H_{4} \wedge A^{(6)} \wedge F^{(6)}+H_{6} \wedge A^{(6)}+\int_{M_{4+1}} \frac{1}{\mu_{4}} H_{2} \wedge A^{(4)} \wedge F^{(4)}+H_{4} \wedge A^{(4)} \tag{4.7}
\end{equation*}
$$

The equations of motion of the $R R$ forms are

$$
\begin{align*}
& d H_{2}=-\delta(789) \\
& d H_{4}=-\delta(23456)-F^{(6)} \wedge \delta(789) \\
& d H_{6}=F^{(4)} \wedge \delta(23456)+\frac{1}{\mu_{6}} F^{(6)} \wedge F^{(6)} \wedge \delta(789) \tag{4.8}
\end{align*}
$$

Taking into account

$$
\begin{equation*}
\int_{S^{p+2}} H_{p+2}=\mu_{6-p} N_{6-p} \tag{4.9}
\end{equation*}
$$

for $p \in\{4,6\}$, and substituting in the equations of motion of the $R R$ forms, we obtain

$$
\begin{align*}
L_{\mathrm{cs}} & =-\int d u A_{+}^{(6)} F_{-u}^{(6)}+A_{-}^{(6)} F_{u+}^{(6)}+A_{u}^{(6)} F_{+-}^{(6)}+d v(u \rightarrow v, 6 \rightarrow 4) \\
L_{\mathrm{mix}} & =-\left(F_{+-}^{(6)} \int d v A_{v}^{(4)}+F_{+-}^{(4)} \int d u A_{u}^{(6)}\right) \tag{4.10}
\end{align*}
$$

The equations of motion obtained from the Lagrangian by varying the gauge field $A^{(6)}$ at arbitrary $u$ are given by

$$
\begin{align*}
\frac{V_{S^{5}}}{g_{6}^{2}}\left[\partial_{u}\left(u^{4} F_{u-}^{(6)}\right)+\partial_{-} F_{+-} u^{4}\right]-2 F_{u-} & =0 \\
\frac{V_{S^{5}}}{g_{6}^{2}} \partial_{u}\left(u^{4} F^{(6)}\right)+2 F_{u+}^{(6)}+\frac{V_{S^{5}} u^{4}}{g_{6}^{2}} \partial_{+} F_{-+}^{(6)} & =0 \\
2 F^{(6)}-F_{+-}^{(4)}-\frac{V_{S^{5}} u^{4}}{g_{6}^{2}}\left(\partial_{-} F_{u+}^{(6)}+\partial_{+} F_{u-}^{(6)}\right) & =0 \tag{4.11}
\end{align*}
$$

Similar equations can be obtained by varying $A^{(4)}$, except that we change all the radial dependence from $u^{4}$ to $v^{2}$ and all radial partial derivatives are taken with respect to $v$. As in eq. (2.18) in [1] the effect of the Chern-Simons terms make the $F_{+-}$mode massive. Since we are interested only in the massless modes we can set them to zero and we are left with

$$
\begin{align*}
& \left.F_{u \pm}^{(6)}=\frac{h^{(6)_{ \pm}}}{u^{4}} e^{\left( \pm \frac{2 g_{6}^{2}}{3 V_{S^{5}} u^{3}}\right.}\right) \\
& F_{v \pm}^{(4)}=\frac{h^{(4) \pm}}{v^{2}} e^{\left( \pm \frac{2 g_{4}^{2}}{V_{S^{2}}{ }^{v}}\right)} \tag{4.12}
\end{align*}
$$

The calculation shows that the equations of motion cease to be symmetric between $A^{(6)}$ and $A^{(4)}$ and the gauge fields are displaced away from the intersection region with different dependences on the radial coordinates $u$ and $v$. As a result the theory distinguishes the two radial directions and so we do not get two identical theories for $u>0$ and $v>0$. Therefore enhanced Poincare symmetry should be absent, confirming the analysis in the supergravity limit.

## 5. Conclusions

We have seen that the curious enhancement of Poincare symmetry observed in (1, 2] close to the intersection region of intersecting D5-branes is reproduced in the supergravity limit, although the supergravity solution is only valid over a limited range of the radial variables and in the large $N$ limit. Special attention is also needed in taking the double scaling limit in order to scale to the intersection domain. However, this symmetry enhancement is absent in the T-dual picture in which a D6-brane intersects a D4-brane. From the open string analysis, this manifests itself as different dependences on the radial variables in the displacement of the chiral fermions from the intersection. From the supergravity solution in the near horizon limit angle deficits develop in the brane bulks, which prevent any symmetry enhancement. We have only treated the supergravity limit where $g_{s}$ is small. To consider the strongly coupled picture, the eleventh dimension in M-theory would emerge and we would need an M-theory description of the system in which D6-branes are KaluzaKlein monopoles in M-theory and the D4-branes are M5 branes compactified on a circle. In the near horizon limit of $N$ KK monopoles, the geometry reduces to an ALE space and the transverse four dimensional space can be described by an orbifold $C^{2} / \mathbf{Z}_{\mathbf{N}}$ [8]. M5 branes can be easily embedded in this space. The metric of the system concerned is given by eq. (247) in [8], from which we conclude that the absence of symmetry enhancement carries over into the strong coupling limit. Nonetheless, it is important to note that the chiral fermion at the intersection gains mass and is displaced away from the intersection even in the T-dual picture. This is exploited in [10, 11] for chiral symmetry breaking.

Finally, we note that the symmetry enhancement disappears as soon as the D5/D5 system is compactified on a circle since the two sets of D5 branes are no longer indistinguishable. This is T-dual to the D6/D4 system on a circle in the D6 world-volume. The analysis here suggests that compactification alone destroys the symmetry enhancement.

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## References

[1] N. Itzhaki, D. Kutasov and N. Seiberg, I-brane dynamics, JHEP 01 (2006) 119 hep-th/0508025.
[2] H. Lin and J.M. Maldacena, Fivebranes from gauge theory, Phys. Rev. D 74 (2006) 084014 hep-th/0509235).
[3] S.G. Naculich and H.J. Schnitzer, Level-rank duality of untwisted and twisted D-branes, Nucl. Phys. B 742 (2006) 295 hep-th/0601175.
[4] M.B. Green, J.A. Harvey and G.W. Moore, I-brane inflow and anomalous couplings on D-branes, Class. and Quant. Grav. 14 (1997) 47 hep-th/96050333.
[5] C.G. Callan Jr., J.A. Harvey and A. Strominger, Supersymmetric string solitons, hep-th/9112030.
[6] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, Supergravity and the large-N limit of theories with sixteen supercharges, Phys. Rev. D 58 (1998) 046004 hep-th/9802042.
[7] J.D. Edelstein, L. Tataru and R. Tatar, Rules for localized overlappings and intersections of p-branes, JHEP 06 (1998) 003 hep-th/9801049.
[8] D.J. Smith, Intersecting brane solutions in string and M-theory, Class. and Quant. Grav. 20 (2003) R233 hep-th/0210157.
[9] A. Hashimoto, Supergravity solutions for localized intersections of branes, JHEP 01 (1999) 018 hep-th/9812159.
[10] E. Antonyan, J.A. Harvey, S. Jensen and D. Kutasov, NJL and QCD from string theory, hep-th/0604017.
[11] E. Antonyan, J.A. Harvey and D. Kutasov, Chiral symmetry breaking from intersecting $D$-branes, hep-th/0608177.


[^0]:    ${ }^{1}$ The supergravity solution in this limit can be obtained directly by dimensional reduction of the solution of an M5 brane embedded in an ALE space, which is the near-core limit of the KK monopole [8, 8].

